On the Economics of Carbon Sequestration in Forest Sites of Different Productivity

Matti Laukkanen
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Tiedekunta/Osasto Fakultet/Sektion – Faculty Faculty of Agriculture and Forestry	Laitos/Institution– Department Department of Economics and Management

Tekijä/Författare – Author
Laukkanen Matti Taneli

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Tiivistelmä/Referat – Abstract

This study analyses the effects of carbon subsidy schemes on the optimal forest management in forest sites of different productivity. Forest stand level analysis shows the changes in the optimal stand management due to carbon subsidies. Market level analysis evaluates the market level implications of mutual and unilateral carbon subsidy policies and their effects outside the policy area.

In the first chapter, we study the effects of carbon subsidies on a forest stand level. The results show that carbon subsidies lengthen the optimal rotation period, increase the annual timber output and increase the amount of CO_2 sequestered in the forest stand. A sufficiently high carbon price leads to forest conservation. All the effects are stronger in the forest of poor productivity.

The market level analysis presents an age-class structured model with an endogenous timber price and alternative land use. The numerical examples show that, in addition to the effects shown in the stand-level analysis, carbon subsidies encourage afforestation. An increase in the annual timber output may lead to a lower stumpage price. Unilateral policies may lead to an increase in timber output inside the policy, which decreases the timber price and result in deforestation outside the policy. As a sufficiently high carbon price leads to forest conservation, timber price increases and results in afforestation and decrease carbon emissions outside the policy. The results are in contradiction with the common hypothesis that increasing carbon sequestration in forests by unilateral policy would inevitably lead to an increase in carbon emissions outside the policy area.

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Tiivistelmä/Referat – Abstract

Pro gradu -työssäni tutkitaan hiilitukien vaikutuksia metsänhoitoon tuottavissa ja heikosti tuottavissa metsissä. Metsikkötason tarkastelu osoittaa hiilitukijärjestelmän aiheuttaman muutoksen optimaalisessa metsänhoidossa. Markkinatason analyysiosiossa arvioidaan yhteisen ja yksipuolisen hiilitukijärjestelmän markkinavaikutuksia tukialueen sisällä ja sen ulkopuolella.

Työn ensimmäisessä osassa tarkastellaan hiilitukien vaikutuksia metsikkötasolla. Tulokset osoittavat, että hiilituet pidentävät optimaalista kiertoaikaa, kasvattavat vuotuista hakkuumäärää sekä lisäävät sidotun hiilidioksidin määrää metsikössä. Kun hiiliyksikön hinta on tarpeeksi korkea, on taloudellisesti kannattavinta jättää metsikkö kokonaan kaatamatta. Hiilituet vaikuttavat voimakkaammin optimaaliseen metsänhoitoon heikosti tuottavalla kasvupaikalla verrattuna tuottavaan kasvupaikkaan.

Seuraavassa osassa hiilitukien vaikutusten tarkastelua laajennetaan markkinatasolle. Markkinatason malli muodostuu endogeenisesta kantohinnasta, useasta metsämaa-alueesta ja vaihtoehtoisesta maankäyttömuodosta. Tulokset osoittavat, että metsikkötasolla osoitettujen vaikutusten lisäksi hiilituet kannustavat metsittämiseen ja voivat laskea kantohintaa, koska puun tarjonta lisääntyy. Yksipuolisissa tukijärjestelmissä laskenut kantohinta voi aiheuttaa metsäkatoa tukijärjestelmän ulkopuolella. Korkea hiiliyksikön hinta johtaa metsänsuojeluun ja vähentynyt tarjonta kannustaa metsittämiseen myös tukijärjestelmän ulkopuolella. Tulokset ovat ristiriidassa yleisen oletuksen kanssa, jonka mukaan hiilensidontaohjelmat johtavat väistämättä hiilipäästöjen kasvuun ohjelman ulkopuolella.

Avainsanat – Nyckelord – Keywords
Hiilensidonta, Hiilivuoto, Faustmannin malli, Ikäluokkarakenteinen malli, Metsämarkkinat, Metsätalous

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1 Introduction

Forestry plays a key role in climate change mitigation and decreasing the amount of carbon dioxide (CO_2) in the atmosphere (IPCC 2019). The most efficient way to manage carbon emissions is to set a price for units of carbon through taxation or purchasable permits through an emissions trading scheme (Baumol 1972, Hagmann et al. 2019). A way for polluters to compensate for their emissions is to introduce forest carbon offsets into the trading scheme, as seen in California and New Zealand (Nurmi and Ollikainen 2019). In both programs, the price of carbon is represented through carbon subsidies and taxes that enter the forest owner's optimization problem (see e.g. Van Kooten et al. 1995).

Currently, carbon pricing mechanisms applied and initiatives under consideration cover a total of 20,1% of global greenhouse gas emissions (World bank 2019). The regulation on Land Use, Land Use Change and Forestry -sector (LULUCF) by European Union in 2018 sets the accounting methods of CO_2 sinks and emissions of the member states (EU 2018/841). The regulation states that the LULUCF-sector of a member state should not be a source of carbon emissions, which brings up the question of the optimal forestry and land-use management practices when carbon sequestration has a value. As the economic viability of storing carbon in a forest depends on its growth, end-products, and non-timber ecosystem services provided, the question of where to increase the carbon storage is not trivial.

Hartman (1976) extended the classic stand-level model by Faustmann (1849) with non-timber amenity values, leading the way for natural resource economists to value forestland in various ways other than sole timber production. Where Hartman considered the external services as a function of the stand age, the model in Van Kooten et. al. (1995) presents the utility of storing carbon in the model as function of the change in stand biomass. The study, among various other economic papers on single-stand optimization problem with carbon pricing (e.g. Hoen and Soldberg 1997, Gong and Kriström 1999, Caparrós 2003), consider the carbon price as a subsidy-based instrument. Recall that when carbon storages are a commodity with a market price, the forest owner's problem is to maximize the net profits from timber production and carbon subsidies.

Under all stand-level analyses with an exogenous timber price lies the axiom of a normal forest, i.e. the timber flow is even over time. This assumption was challenged by Mitra-Wan tree farm model

(Mitra and Wan 1985), which presented a market-level economic problem where, with endogenous timber price and discounting, an optimal harvest leads to a cyclical solution instead of a steady state. Later, Salo and Tahvonen (2002) proved analytically that the cycles occur due to a discrete time model. In Salo and Tahvonen (2004), it is shown that in the presence of alternative land-use in the optimal solution, the cycles vanish and the age-class allocation converges towards a normal forest. Carbon sequestration is introduced into the model by Cunha-e-Sá et al. (2006, 2013), Akao (2011), and Tahvonen and Rautiainen (2017). In Tahvonen and Rautiainen (2017), they prove the existence of optimal solution where a part of the forestland is allocated purely for carbon storage purposes, i.e. a high enough carbon price leads to forest conservation.

Currently, the existing carbon pricing schemes worldwide vary in their pricing mechanisms and the accounting methods. In economic literature, an optimal price for carbon is characterized as a marginal social cost of carbon, i.e. the amount of economic loss from releasing one unit of carbon into the atmosphere (Cai et al. 2015, Nordhaus 2017). Nordhaus (2017) estimates that the global social cost of carbon in 2050 with the current policies would be \$50 - \$250 per ton of CO_2 in 2010 US dollars, depending on the interest rate. Cai et al. (2015) use a model with a climate tipping point and estimate the social cost of carbon to be \$316 - \$814 per ton in 2100.

Although the variability and uncertainty with future cost estimations are high, there is a consensus among climate economists that the current carbon prices have to increase substantially in the upcoming decades (Nurmi and Ollikainen 2019). Stand-level studies have shown that an increase in carbon price may lengthen the optimal rotation period, increase annual timber output and ultimately lead to forest conservation (see e.g. Olscheswski and Benítez 2010, Akao 2011, Assmuth et al. 2018). Market-level studies based on the Mitra-Wan tree farm model find that an increase in the carbon price will lengthen the optimal rotation period, lead to afforestation and thus increase the agricultural land rent (Cunha-e-Sá et al. 2013, Tahvonen and Rautiainen 2017.) None of the studies based on the Mitra-Wan tree farm model include multiple land classes and carbon subsidies simultaneously. This would allow the study of the unilateral carbon sequestration program's effects in areas outside the program, i.e. Carbon leakage.

Existing market-level studies on carbon leakage are performed through theoretical analyses, econometric studies and general and partial equilibrium modeling (see e.g. Sohngen et al. 1999, Murray et al. 2004, Nepal et al. 2013, Harstad and Mideksa 2017). Murray et al. (2004) provide a general equilibrium model to show that the carbon leakage in U.S. may range from 10% to 90%, depending on the sequestration activity and region. Similarly, Gan and McCarl (2007) estimate that, on a global scale, 42% - 95% of the positive environmental gains are offset elsewhere. Both studies conclude that the magnitude of carbon leakage depends on the demand elasticity. In addition, all these studies assume that carbon sequestration programs *decrease* the timber supply, which will be satisfied by increasing harvests elsewhere, leading to carbon leakage. The assumption fails to consider a) the stand-level studies showing that carbon subsidies may increase the long run timber supply (e.g. Assmuth et al. 2018) and b) that an increase in profitability due to carbon subsidies may lead to afforestation in commercial forestry (Cunha-e-Sá et al. 2013, García et al. 2018).

This thesis aims to challenge the above-mentioned assumptions of forest-sector carbon leakage. In chapter two, we present both analytical and numerical results for the van Kooten et al. (1995) stand-level model with poor and fertile forest stands. Chapter three presents the Mitra-Wan tree farm model with carbon subsidies by Tahvonen and Rautiainen (2017), expanded with multiple land classes (Salo and Tahvonen 2002). Numerical results analyze first the effects of a mutual subsidy scheme with two land classes and second the effects of a unilateral subsidy scheme and its market implications. Poor and fertile forestland are compared throughout the analysis to evaluate the role of forest productivity in carbon sequestration programs.

2 Carbon Sequestration in a Stand-Level Optimization Problem

This chapter presents the traditional Faustmann model (see e.g. Samuelson 1976) for calculating the optimal forest rotation length for a single stand when the forest's carbon storage is taken into account (E.g. Van Kooten et al. 1995). The price per ton of carbon (CO_2) is denoted by P_c, which can represent the social cost of carbon in a society or a price of carbon dioxide in an emission trading scheme. Carbon storages are interpret as positive externalities that are subsidized to reach their socially optimal level. For the landowner, the problem is to maximize the net income from both timber production and carbon sequestration subsidies. It is assumed that the initial state is bare land. The model does not take natural regeneration or thinnings into account and yields somewhat unrealistically short rotation periods.

2.1 Model

The stand volume ($m^3\ ha^{-1}$) is given as a function F of stand age t. The function satisfies the conditions

$$F \in C^3, F(0) = 0, F'(0) = 0, F(t) > 0 \; and \; F'(t) > 0 \; for \; all \; t > 0, \; F'(t) \to 0 \; and \; F \to \hat{F} \; as \; t \to \infty,$$

$$F'' > 0 \text{ for } 0 < t < \hat{t}, \quad F'' < 0 \text{ for } t > \hat{t} \text{ and } \frac{F''}{F'} \text{ is decreasing in } t, \tag{A1}$$

where $\hat{t}$ notes the culmination age where the stand growth is fastest and $\hat{F}$ is the volume where the stand growth has reached its maturity. An example of a biological growth function that satisfies A1 is written

$$F(t) = \alpha_1(1 - e^{-\alpha_2 t})^{\alpha_3},$$

where $\alpha_1 > 0$ is the asymptote, i.e. the level $F(t)$ approaches as $t \to \infty$ and $\alpha_2 > 0$, and $\alpha_3 > 0$ are empirical growth parameters. Figure 1 shows the development of stand volume F in time t. The parameters equal $\alpha_1 = 465, \alpha_2 = 0{,}07$, and $\alpha_3 = 17{,}6$.

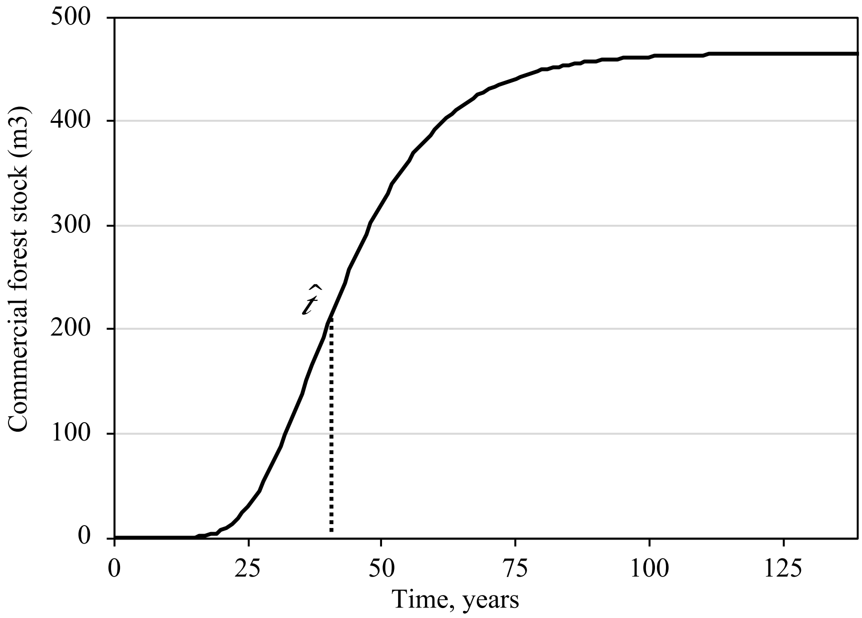


Figure 1. Stand volume function F.

When the stand is young, the function F is increasing and convex and turns to increasing and concave after reaching the culmination age denoted by $\hat{t}$.

The objective of the Faustmann model is to maximize bare land value J_f by optimizing the forest rotation length in t

$$J_f(t) = \frac{-w + pF(t)e^{-rt}}{1 - e^{-rt}} \tag{1}$$

where the stumpage price $p \geq 0$ (€ per m^3) is constant, implying linear utility, $w \geq 0$ denotes regeneration cost per hectare and $r \geq 0$ is the interest rate. The first order condition of (1) is

$$J_f'(t) = pF'(t) - rpF(t) - rJ_f(t) = 0,$$

which can be interpret as follows: it is optimal to clearcut the stand when the marginal value of stand growth $pF'(t)$ equals the interest costs of postponing the harvest $rpF(t)$ and the interest costs of postponing the revenues of all future harvests $rJ_f(t)$. Solving t in $J_f'(t) = 0$ gives us the optimal clearcut age $t*$ and $J_f(t^*)$ is the maximized bare land value of the forest stand.

Let the price of carbon per cubic meter of commercial timber to be denoted by τ and write $\tau = \delta p_c$, where δ is the carbon content of one cubic meter and p_c is the social cost of carbon. The parameter β $(0 \leq \beta \leq 1)$ is the present value of the release of carbon from harvested timber products, i.e. the rate of decay. If $\beta = 1$, all carbon from the harvested timber is released immediately. If $\beta = 0$, carbon is stored forever in the end-products. The economic value of carbon intake of a forest stand over infinite chain of rotations $J_c(t)$, presented similarly by van Kooten (1995), is written as

$$J_c(t) = \frac{\int_0^t \tau F'(s)e^{-rs}ds - \tau\beta F(t)e^{-rt}}{1-e^{-rt}} \tag{2}$$

where $\int_0^t \tau F'(s)e^{-rs}ds$ is the value of the change in carbon net flow and $\tau\beta F(t)e^{-rt}$ is the value of carbon released upon harvest. To combine the optimal timber production and carbon sequestration problem, we write

$$\max_{\{t\geq 0\}} J_f(t) + J_c(t) = \frac{-w + pF(t)e^{-rt}}{1-e^{-rt}} + \frac{\int_0^t \tau F'(s)e^{-rs}ds - \tau\beta F(t)e^{-rt}}{1-e^{-rt}}$$

$\Leftrightarrow$

$$\max_{\{t\geq 0\}} J(t) = \frac{-w + \tau\int_0^t F'(s)e^{-rs}ds + e^{-rt}(p-\tau\beta)F(t)}{1-e^{-rt}}. \tag{3}$$

Using the Leibniz's formula for integrals (see e.g. Kaplan 1973), we can differentiate (3) with respect to *t* and write the optimality condition as

$$J'(t) = \frac{[p+(1-\beta)\tau]F'(t) - r[(p-\tau\beta)F(t) + J(t)]}{1-e^{-rt}} = 0, \tag{4}$$

where the denumerator $1 - e^{-rt}$ is always positive for any finite $t > 0$ and $r > 0$. The first order condition shows that it is optimal to harvest the stand when the value of marginal growth $[p + (1-\beta)\tau]F'(t)$ equals the interest costs r of postponing the clearcut $(p - \tau\beta)F(t)$ and postponing the revenues of all the future harvests $J(t)$. The finiteness and uniqueness of the optimal rotation is proven in Tahvonen and Rautiainen (2017). If $\beta = 1$ and $r > 0$, a positive carbon price lengthens the rotation period and shortens it if $\beta < 1$, *r = 0* and *w > 0*. In this thesis, we use β values of 1 and 0.822 when *r = 0.03* and 1 and 0.933 when *r = 0.01.*

Proposition 1. *Given* $F(t) = \alpha_1 f(t)$*, where* $f(t)$ *satisfies A1, the maximum volume of the stand* α_1 *decreases the optimal rotation period t* when r > 0 and w > 0.*

Proof. Write (10) as

$$H(t,\alpha_1) = \frac{[p+(1-\beta)\tau]\alpha_1 f'(t)}{1-e^{-rt}} - \frac{r(p-\tau\beta)\alpha_1 f(t)}{1-e^{-rt}} - r\frac{-w}{(1-e^{-rt})^2} - \frac{r\tau\alpha_1\int_0^t[f'(t)e^{-rs}]ds+e^{-rt}(p-\tau\beta)\alpha_1 f(t)}{(1-e^{-rt})^2} = 0. \tag{5}$$

Dividing by α_1 yields

$$H(t,\alpha_1) = \frac{[p+(1-\beta)\tau]f'(t)}{1-e^{-rt}} - \frac{r(p-\tau\beta)f(t)}{1-e^{-rt}} - r\frac{\frac{-w}{\alpha_1}}{(1-e^{-rt})^2} - \frac{r\tau\int_0^t[f'(t)e^{-rs}]ds+e^{-rt}(p-\tau\beta)f(t)}{(1-e^{-rt})^2} = 0. \tag{6}$$

Differentiate (12) with respect to α_1 to get

$$\frac{\partial H}{\partial \alpha_1} = -r\frac{\frac{w}{a1^2}}{(1-e^{-rt})^2} < 0 \text{ for all } r > 0,\ w > 0.$$

Given that $J'(t^*) = 0$, we write

$$\frac{\partial H}{\partial t} = \frac{[p+(1-\beta)\tau]\alpha_1 f''(t) - r[(p-\tau\beta)]\alpha_1 f'(t)}{(1-e^{-rt})},$$

As shown in Tahvonen & Rautiainen (2017), $\frac{\partial H}{\partial t}$ is negative when $f(t)$ satisfies A1. By using implicit function theorem we get

$$\frac{\partial t^*}{\partial \alpha_1} = -\frac{-r\frac{\frac{w}{a1^2}}{(1-e^{-rt})^2}}{(1-e^{-rt})^{-1}[[p+(1-\beta)\tau]\alpha_1 f''(t) - r[(p-\tau\beta)]\alpha_1 f'(t)]} < 0 \text{ for all } r > 0,\ w > 0. \tag{7}$$

Thus, with a positive interest rate and regeneration cost, the optimal rotation is a decreasing function of the maximum stand volume. As the maximum stand volume increases, the optimal rotation period decreases. With lower values of α_1, the optimal rotation is longer. ■

2.2 Numerical results

Let us compare the effect of a positive carbon price on fertile and poor forest sites. The growth function parameters are derived from the data of Motti simulator developed by the Natural Resource Institute of Finland (Luonnonvarakeskus, 2019) and represent the growth of pure Norway spruce stand in the Southern Finland (fertile forest) and pure Scots pine stand in Lapland (poor forest).

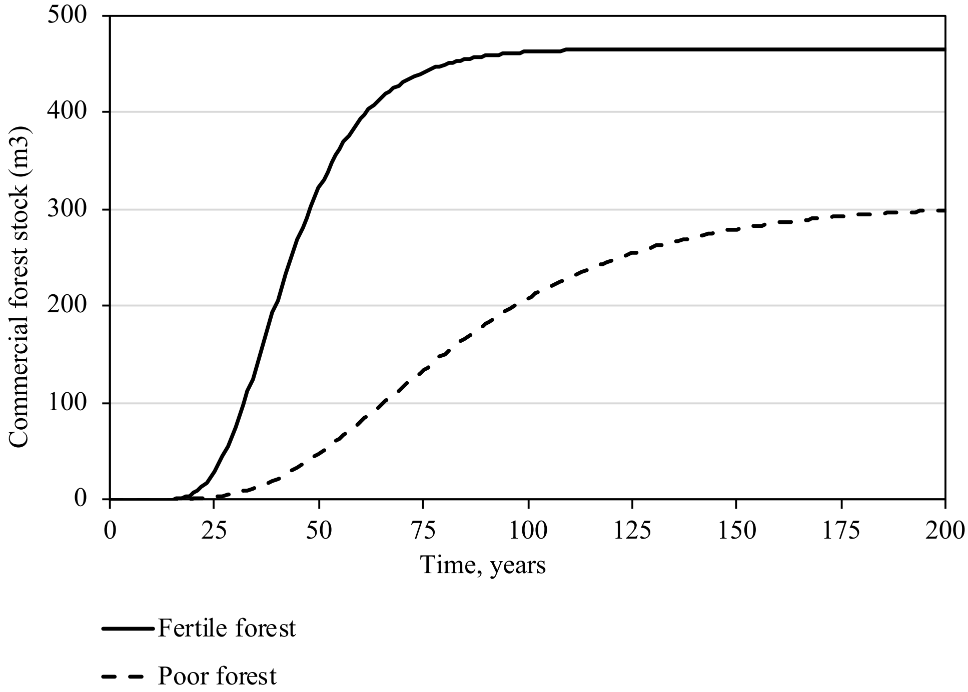


Figure 2. Fertile forest and poor forest stand volume functions.

Note. The parameter values are $\alpha_1 = 465, \alpha_2 = 0{,}07$, and $\alpha_3 = 17{,}6$ for the fertile forest and $\alpha_1 = 305, \alpha_2 = 0{,}04$, and $\alpha_3 = 7{,}2$ for the poor forest.

Stumpage price is 45€ at the fertile site and 25,50€ at the poor site, derived from the end-product averages of Motti simulator. Regeneration costs are 1500€ and 300€ respectively, assuming that they are lower at the poor site due to different regeneration practices. The calculations are made using Maple-software (version 2019).

The numerical results in Table 1 show that the optimal rotation period is shorter in the fertile forest stand and longer in the poor forest stand (proposition 1). A positive carbon price lengthens the rotation period and increases the amount of carbon sequestered in both sites. Note that we use the discounted value of carbon tons sequestered in the forest, as there is a time preference in the release of carbon into the atmosphere. With moderate CO_2 prices the annual timber production increases, as shown previously in detailed models by optimized thinning by Pihlainen et al. (2014) and Pohjola and Valsta (2007). This indicates that the economically optimal rotation period without carbon subsidies is shorter than the rotation implying Maximum Sustainable Yield (later MSY). The result underlines the importance of economic optimization, as the result contradicts with some ecological studies showing that an increase in carbon sequestration leads inevitably to a decrease in timber harvests (cf. Kaipainen et al. 2004, Liski et al. 2001). A sufficiently high carbon price results in a scenario where the whole stand is left unharvested (Figures 3 a,b). This suggests that the optimal management regime of the forest changes from clearcut to forest conservation. In all examples, the poor forest is left under conservation with lower price of carbon than the fertile forest.

The cost of an additional ton of CO_2 stored is calculated by comparing the losses of timber net revenues to the additional tons of carbon sequestered (Table 1). With lower prices of CO_2, the cost per ton of an additional CO_2 stored is higher at the poor site. Tahvonen and Rautiainen (2017) prove analytically that a positive price of carbon lengthens the rotation period when $r > 0$. Our results show that the price of carbon lengthens the optimal rotation period more at the poor forest stand due to the lower slope of the growth function at t^*, i.e. slower forest growth leads to a greater change in the optimal rotation. When $r > 0$ and $\tau > 0$, longer optimal rotation period leads to lower timber income due to discounting. Higher decrease in the net present value of timber production leads to a higher cost of additional CO_2 stored. When the CO_2 price is high enough to allocate the poor forestland solely for carbon storage purposes, an additional ton of CO_2 stored is more costly to the forest owner in the fertile site. This suggests that the initial BLV is higher at the fertile site and the loss of income due to increased carbon sequestration is higher with higher values of τ. (Table 1.)

If $\beta < 1$, the optimal rotation period is shorter and a part of the carbon remains in timber products and decreases the cost of additional CO_2 stored. The effect of $\beta < 1$ on the optimal rotation period is stronger in the poor forest stand (Figure 3a). This is due to a lower slope of $F(t)$ in the poor forest stand at t^* and results in relatively higher decrease in the cost of additional CO_2 stored. However, when the forest stand is under conservation, $\beta = 1$ results in lower cost of additional CO_2 stored than $\beta < 1$ due to a lower initial amount of stored CO_2 in the forest stand. When the carbon units stored in timber products are accounted, the baseline carbon storage is higher and the maximum additional sequestration lower, increasing the cost of additional carbon units stored when the stand is under conservation. These results underline the importance of the method used for carbon accounting, since it has different effects on the optimal solution depending on the stand productivity.

Forestry becomes more profitable for forest owners at both sites when a carbon subsidy -scheme is applied. From the social planner’s perspective, carbon subsidies can be very costly and thus the scheme should be based on additionality (Table 1).Given the future carbon price range estimations by Cai et al. (2015) and Nordhaus (2017), most scenarios presented in our examples would lead to forest conservation (Figures 3a,b). However, we can conclude that the stand-level analysis leaves much to be questioned due to the simplicity of the model used. In the next chapter, we extend the analysis to a market-level optimization problem where timber price is endogenous and alternative land use is taken into account.

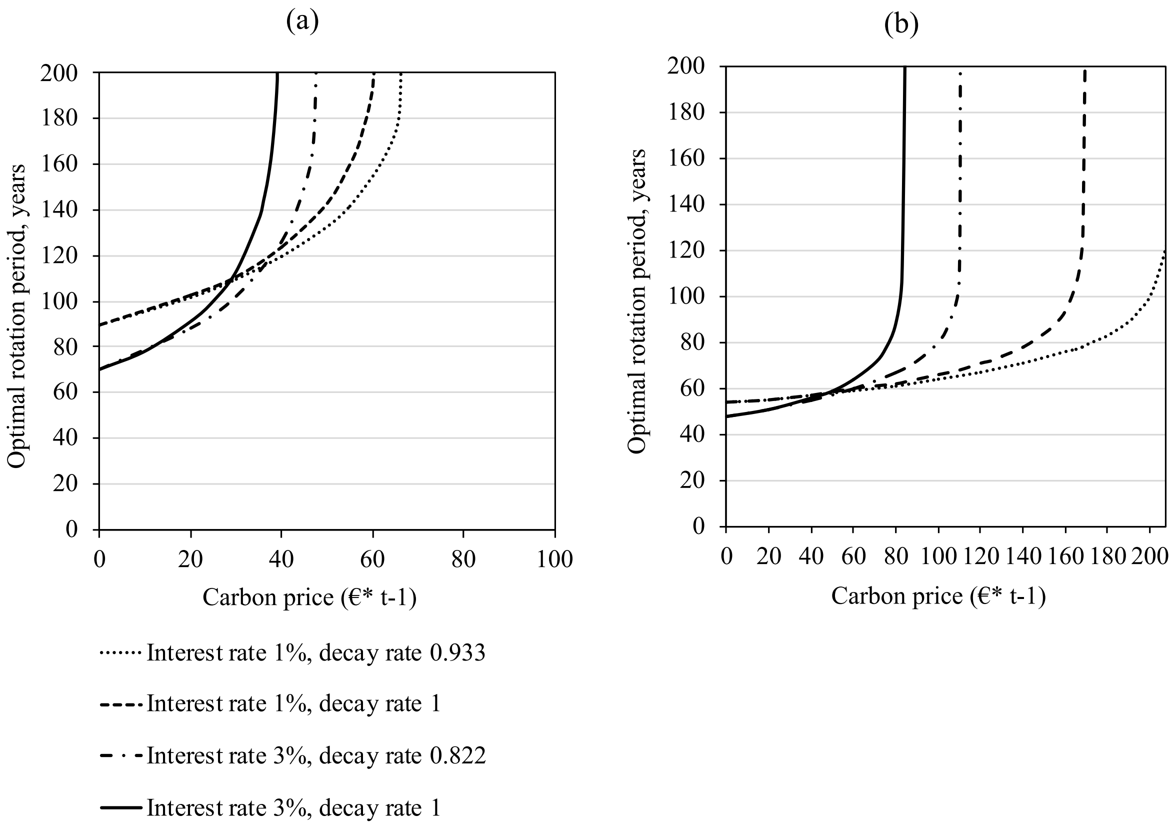


Figure 3. Optimal rotation periods with different interest rates and rates of decay for (a) the poor site and (b) the fertile site.

Table 1. Carbon subsidies in poor and fertile forest stand.

	(A)	(B)	(C)	(D)	(E = C + D)	(F = D - A * G_0)	(G = D / A)	(H)	(I = C_0 - C / G - G_0)
Site	CO_2 price (€* t^{-1})	Rotation period (years)	NPV of timber income (€*ha^{-1})	Total subsidies (€*ha^{-1})	Total Revenue (€*ha^{-1})	Subsidies based on additionality (€*ha^{-1})	Present value of carbon stored (CO_2 t*ha^{-1})	Annual wood production (m^3*ha^{-1})	Cost of additional CO_2 stored (€*tCO_2^{-1})
Poor forest	0	70	72	0	72	0	9,8	1,7	0
	20	91	10	310	320	114	15,5	2	10,9
	40	Inf.*	-300	1022	722	630	25,6	0	23,6
	60	Inf.	-300	1534	1234	945	25,6	0	23,6
	80	Inf.	-300	2045	1745	1260	25,6	0	23,6
Fertile forest	0	48	2253	0	2253	0	31,9	6,3	0
	20	51	2192	753	2945	115	37,7	6,5	10,3
	40	56	1965	1805	3770	529	45,1	6,6	21,7
	60	64	1427	3345	4772	1429	55,7	6,4	34,6
	80	88	-35	6090	6055	3536	76,1	5,2	51,7

Note: Parameter values are r = 0.03, β = 1.

* marks the situation when forest is allocated solely for carbon storage purposes.

3 Carbon Sequestration in a Market-Level Optimization Problem

This chapter presents the market-level age-class forestry model studied in Mitra and Wan (1985, 1986) extended to include multiple land classes, carbon storage, and land allocation between forestry and an alternative land use, such as agriculture (Salo and Tahvonen 2002, 2004; M.A. Cunha-e-Sá et al. 2013, Piazza and Roy 2015, Tahvonen and Rautiainen 2017). The social planning problem is similar to Tahvonen and Rautiainen (2017), but expanded with multiple land classes (Salo and Tahvonen 2002).

3.1 Model

Different forest land areas are divided into classes $i = 1,\ldots,h$. The age classes are denoted by $s = 1,\ldots,n$, where all stands aged n or older are allocated to class n. Note that in the presence of carbon price, trees in class n have economic value and may remain unharvested. The area allocated to age-class s in forest land class i at the beginning of period t is x_{ist}, $t = 0,1\ldots$. The area allocated for alternative land use is denoted by y_{it} for each forest land class i. Total land area in each class i equals one. The volume of harvestable timber per hectare in land class i and age class s is f_{is}, with assumptions

$f_{i1} \geq 0, f_{is} < f_{i,s+1}$ for $s = 1,\ldots,n-2$, and $f_{i,n-1} = f_{in}$.

The per period harvest of all forest land classes at the end of the period t is

$$c_t = \sum_{i=1}^{h} \sum_{s=1}^{n-2} f_{is}\left(x_{ist} - x_{i,s+1,t+1}\right) + f_{in}\left(x_{int} + x_{i,n-1,t} - x_{in,t+1}\right). \tag{8}$$

Note that because stand volume does not grow in age classes older than $n - 1$, and unharvested land area of age n stays in that age class, the timber yield from harvesting the two oldest age classes equals

$$f_{in}\left(x_{int} + x_{i,n-1,t} - x_{in,t+1}\right).$$

The price per ton of carbon and the carbon content of one m^3 of timber is denoted by τ and δ, respectively. The total carbon volume C in all forest classes at the beginning of period t equals

$$C = \delta\left[\sum_{i=1}^{h}\sum_{s=1}^{n} f_{is}x_{ist}\right]. \tag{9}$$

The parameter β $(0 \leq \beta \leq 1)$ is the present value of the release of carbon from harvested timber products as in the stand level model. The per period net carbon inflow in all living trees and timber products equals

$$\tau\delta\sum_{i=1}^{h} Q_t = \tau\delta\sum_{i=1}^{h}\left[\sum_{s=1}^{n} f_{is}\left(x_{is,t+1} - x_{ist}\right) + (1-\beta)c_{it}\right] \tag{10}$$

The utility from timber consumption U $(U' > 0,\ U'' < 0)$ is derived from the inverse demand function $D_c(c_t)$ and can be given as $U(c_t) = \int_0^{c_t} D_c(c)dc$. Similarly, the utility from the alternative use of land W $(W' > 0,\ W'' < 0)$ is derived from $D_y(y_t)$ and written $W(y_t) = \int_0^{y_t} D_y(y)dy$. The discount factor is denoted by b $(0 < b < 1)$ and the social maximization problem is written

$$\max_{\{x_{ist}, i=1,\dots,h, s=1,\dots,n, t=0,1,\dots\}} V = \sum_{t=0}^{\infty} b^t\left[U(c_t) + \sum_{i=1}^{h} W(y_{it}) + \tau\delta\sum_{i=1}^{h} Q_{it}\right] \tag{11}$$

subject to

$$x_{i,s+1,t+1} \leq x_{ist}, s = 1, \dots, n-2 \text{ and } x_{in,t+1} \leq x_{int} + x_{i,n-1,t}, \tag{12}$$

$$\sum_{s=1}^{n} x_{ist} \leq 1, \tag{13}$$

$$x_{ist} \geq 0, i = 1, \dots, h, s = 1, \dots, n, \tag{14}$$

$$x_{is0} \geq 0, i = 1, \dots, h, s = 1, \dots, n, \text{ and given,} \tag{15}$$

$$y_{it} = 1 - \sum_{s=1}^{n} x_{ist}. \tag{16}$$

The social planner's problem is to maximize the present value of utility from harvesting forests of different age classes, allocating land between forestry and other uses, and storing carbon in living trees or in timber products if $\beta < 1$. If $h = 1$, the problem is similar to that in Tahvonen and Rautiainen (2017). The Lagrangian for the problem is

$$L = \sum_{t=0}^{\infty} b^t \left[U(c_t) + \sum_{i=1}^{h} W(y_{it}) + \tau\delta \sum_{i=1}^{h} Q_{it} + \sum_{i=1}^{h} \lambda_{it}(1 - \sum_{s=1}^{n} x_{is,t+1}) + \right.$$
$$\left. \sum_{i=1}^{h} \sum_{s=1}^{n-2} P_{ist}\left(x_{ist} - x_{i,s+1,t+1}\right) + \sum_{i=1}^{h} P_{i,n-1,t}\left(x_{int} + x_{i,n-1,t} - x_{in,t+1}\right)\right], \quad (17)$$

where $\lambda_{it}, i = 1, \dots, h, t = 0,1, \dots$ and $P_{ist}, i = 1, \dots, h, s = 1, \dots, n, t = 0,1, \dots$ are the Lagrangian multipliers. Multipliers λ_{it} represent the value of marginal land in various land classes and p_{ist} represents the marginal value of forest land in age class s of the land class i in the beginning of period t+1. For notation purposes, let $F_s = \sum_{i=1}^{h} f_{is}$ denote the total forest growth in age class s.

The Kuhn-Tucker conditions for $t = 0,1, \dots$ are

$$b^{-t} \frac{\partial L}{\partial x_{i,1,t+1}} = bF_1 U'(c_{t+1}) - b \sum_{i=1}^{h} W'\left(y_{i,t+1}\right) - \sum_{i=1}^{h} \lambda_{it} + b \sum_{i=1}^{h} P_{i,1,t+1} + \tau\delta F_1(1 - b\beta) \leq 0, \quad (18)$$

$$b^{-t} \frac{\partial L}{\partial x_{i,s+1,t+1}} = -F_s U'(c_t) + bF_{s+1} U'(c_{t+1}) - b \sum_{i=1}^{h} W'\left(y_{i,t+1}\right) - \sum_{i=1}^{h} \lambda_{it} - \sum_{i=1}^{h} P_{ist} +$$
$$b \sum_{i=1}^{h} P_{i,s+1,t+1} + \tau\delta[F_{s+1}(1 - b\beta) - F_s(1 - \beta)] \leq 0 \text{ for } s = 1,\dots,n-2, \quad (19)$$

$$b^{-t} \frac{\partial L}{\partial x_{i,n,t+1}} = -F_n U'(c_t) + bF_n U'(c_{t+1}) - b \sum_{i=1}^{h} W'\left(y_{i,t+1}\right) - \sum_{i=1}^{h} \lambda_{it} - \sum_{i=1}^{h} P_{i,n-1,t} +$$
$$b \sum_{i=1}^{h} P_{i,n-1,t+1} + \tau\delta F_n \beta(1 - b) \leq 0, \quad (20)$$

$$x_{is,t+1} \geq 0,\ x_{is,t+1} \frac{\partial L}{\partial x_{is,t+1}} = 0,\ s = 1, \dots, n, \quad (21)$$

$$P_{ist} \geq 0, P_{ist}\left(x_{ist} - x_{i,s+1,t+1}\right) = 0, s = 1, \dots, n-2;\ P_{i,n-1,t}\left(X_{int} + X_{i,n-1,t} - x_{in,t+1}\right) = 0, \quad (22)$$

$$\lambda_{it} \geq 0,\ \lambda_{it}(1 - \sum_{s=1}^{n} x_{is,t+1}) = 0. \quad (23)$$

Using the Lagrange method in dynamic optimization is presented in detail by Chow (1997). Salo and Tahvonen (2002) provide a full proof for the existence of a cyclical stationary state for any number of land and age classes when all land is allocated to forestry. If it is optimal to allocate a share of the land for the alternative use, the cycles vanish and the forest age class allocation move towards a steady state, as proven in Salo and Tahvonen (2004) with one land class. M.A. Cunha-e-Sá et al. (2013) and Tahvonen and Rautiainen (2017) show the existence of a stationary state when carbon subsidies are applied but for a model with one land class only.

Stationary cycles

The Faustmann rotation age of a forest land class i is denoted by m_i $(1 < m_i < n_i)$ and satisfies

$$\frac{b^{m_i} f_{im}}{(1-b^{m_i})} \geq \frac{b^s f_{is}}{(1-b^s)} \text{ for } s = 1, \ldots, n,. \qquad (24)$$

If the Faustmann rotation age is the optimal solution for the problems (12) - (15) when $x_{i0} = x_i$, the age class structure x_i for every land class type i has the property $x_i \in S$, and $x_{is} = 0$ for $s_i = m_i + 1, \ldots, n_i$, a forest is called an Optimal Faustmann Forest (OFF). An OFF is called an interior Optimal Faustmann Forest if $x_{is} > 0$ for $s = 1, \ldots, m_i$, $i = 1, \ldots, h$. Salo and Tahvonen (2002) proves that besides the normal forest solution when $x_i = (1/m_i, \ldots, 1/m_i, 0, \ldots, 0)$, there are other OFFs with uneven land allocation structures between land classes which lead to cyclical timber output. Mitra and Wan (1986) show that with one land class and a 0 % interest rate the optimal solution converges towards a normal forest state. This may not hold with multiple land classes, as proven in Salo and Tahvonen (2002). If land classes i and j have optimal rotation periods $m_i \neq m_j$ with a common divisor m' greater than 1, there is a continuum of stationary age-class structures in addition to the normal forest state (Salo and Tahvonen, 2002). Numerical results in Salo & Tahvonen (2002) show that, with $m' > 1$ and no discounting, the solution converges towards a cyclical stationary state with an even total timber flow. With discounting, the cycle radius in the solution increases and the total timber flow turns cyclical. When $h=1$ and the optimal solution has land in alternative use, the forestland converges towards a normal forest structure also under discounting (Salo and Tahvonen, 2004). Note that the model in Salo and Tahvonen (2002) with multiple land classes does not include alternative land use. Cyclicality of the steady state with multiple land classes and alternative land use has not been studied in literature before.

Due to the strictly concave utility function and diminishing marginal returns, a cyclical timber outflow yields utility losses. Salo and Tahvonen (2004) describe three possibilities for smoothening the cyclical age-class structure towards a normal forest state. It is possible to clearcut the stand in an age class before or after the optimal rotation age class. In addition, it is possible to smoothen the cycle by postponing replanting and leaving part of the land bare. However, with a positive discount rate, the marginal costs from smoothing the harvesting cycle are always positive and exceed the marginal utility from smoother timber flow when the solution is arbitrarily close to the normal forest (Salo and Tahvonen 2004). This explains why the optimal solution is cyclical when all land is allocated to forestry. In the presence of an alternative land use, smoothing the cycles by land allocation is possible without utility losses (Salo and Tahvonen 2004).

Due to the complexity of the mathematical analysis of the model, we focus on the numerical analysis with two land classes, i.e. $h = 2$. Parameters for harvestable timber volume f_{is} represent the poor and fertile forest volume functions in the stand-level chapter. First, we examine the stationary state solutions without applying carbon subsidies. Both forestlands supply timber to the same markets and have a possibility to allocate land for alternative use. In the second step, we present the first-best policy scenario where both land classes are placed under the same carbon subsidy scheme and next the second-best policy -scenarios, where a unilateral carbon policy is implemented on one of the land classes. We examine the concept of leakage and show how a unilateral subsidy scheme affects the optimal solution of other forests in the market.

3.2 Numerical Results

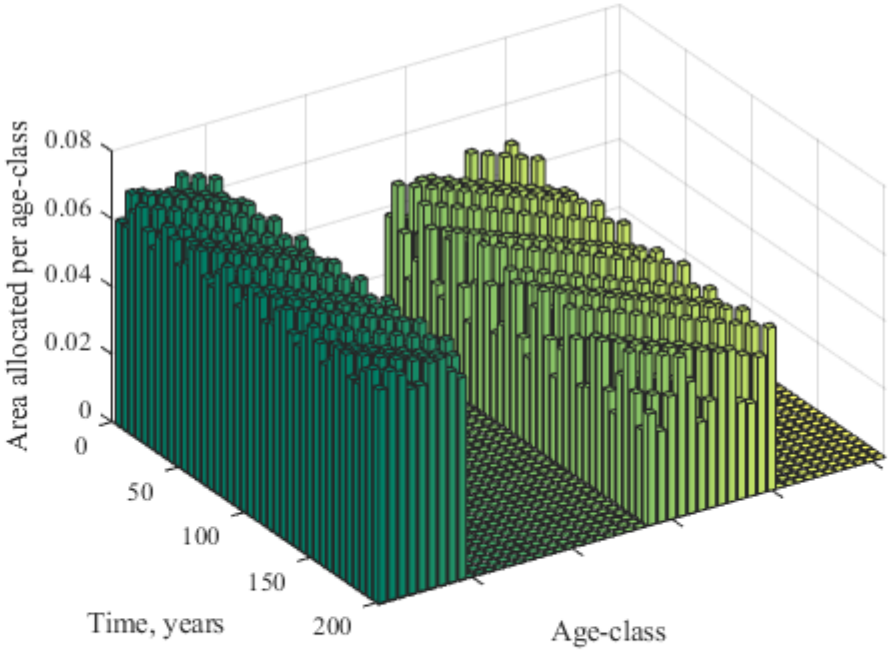
The model is solved as a nonlinear dynamic optimization problem applying AMPL programming language and Knitro optimization software (version 11.1.0). Length of the time horizon is 200 5-year periods. Both forestland classes consist of 24 five-year age classes s and supply timber to the same markets $U(c_t)$, implying that their timber products are perfect substitutes. Age classes are harvested at the end of the 5-year period and the optimal harvesting age is noted in years (i.e. if clearcut is carried out on age class 9, the optimal rotation is 45 years). In addition, we assume that the alternative land uses are independent between land classes, i.e. $W(y_{it}), i = 1..h$.[1]

Examples without carbon policy

Let us first examine the equilibrium stationary state solutions without applying carbon subsidies. Computation shows that, when $r = 0.03$, OFF's for fertile and poor forest are $m_1 = 45$ and $m_2 = 65$, respectively. In Figures 4 a,b it is assumed that r = 0.03 and the initial land allocation x_{is0} is the stationary state solution. The blocks in Figure 4a represent the age class structures of the fertile forest (left side) and the poor forest (right side). The length of the individual bars represent the fraction of land (z-axis) allocated to age class s (x-axis) in time t (y-axis). Both of the age classes follow their OFF's and converge towards a cyclical age-class structure and harvests, resulting in an even total flow of timber (Figure 4b). Land allocation between forestry and alternative land use is cyclical in both land classes (Figure 5). Computation shows that the initial age-class structure does not affect the land allocation in the equilibrium, as shown in Salo and Tahvonen (2004) with one land class and in the absence of land conversion costs.

1) The utility function parameters are $U(c_t) = c^{0.8}, W1(y_t) = 10y^{0.4}, W2(y_t) = 2y^{0.4}$. For forest growth parameters, see Appendix 1.

(a)



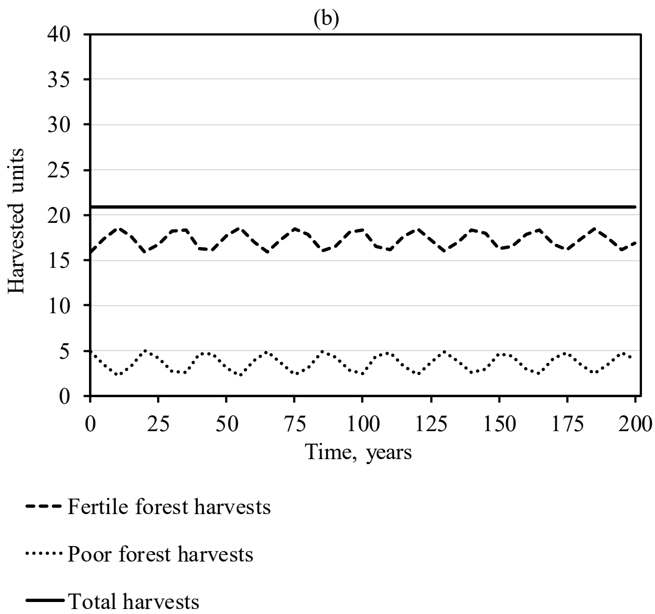


Figure 4a,b. Stationary state (a) age-class structures and (b) harvests.

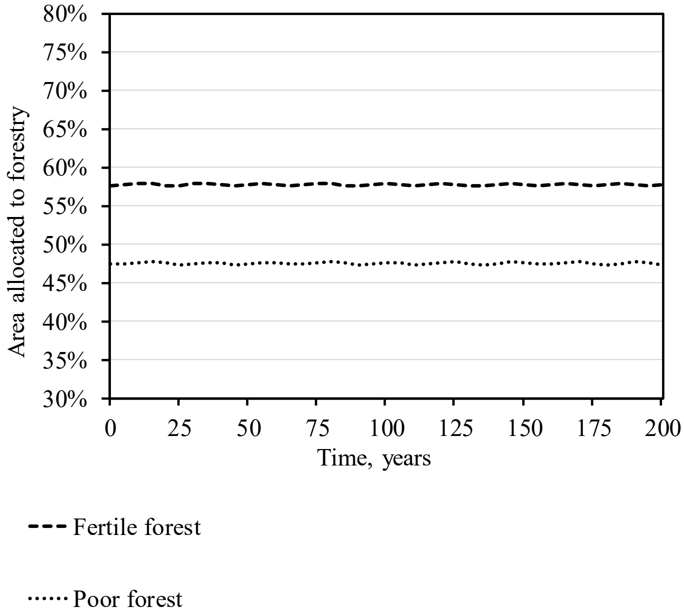


Figure 5. Stationary state land allocation.

The result suggests that, with multiple land classes, it is optimal to allocate land between forestry and alternative land use so that the total timber output converges towards a steady state with an even timber flow. With a 1% interest rate the stationary state harvests and land allocation are less cyclical (Figure 6). The result implies that the cyclicality in harvests of each land class and in land allocation bears utility losses and vanishes as the interest rate approaches zero, as in the examples with one land class (see Salo and Tahvonen 2002).

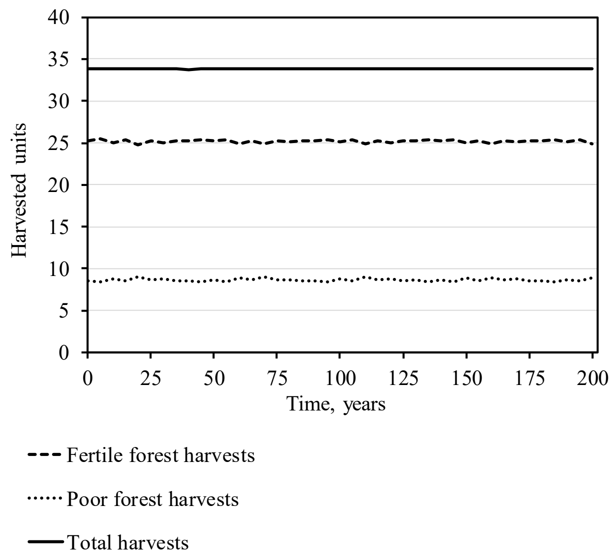
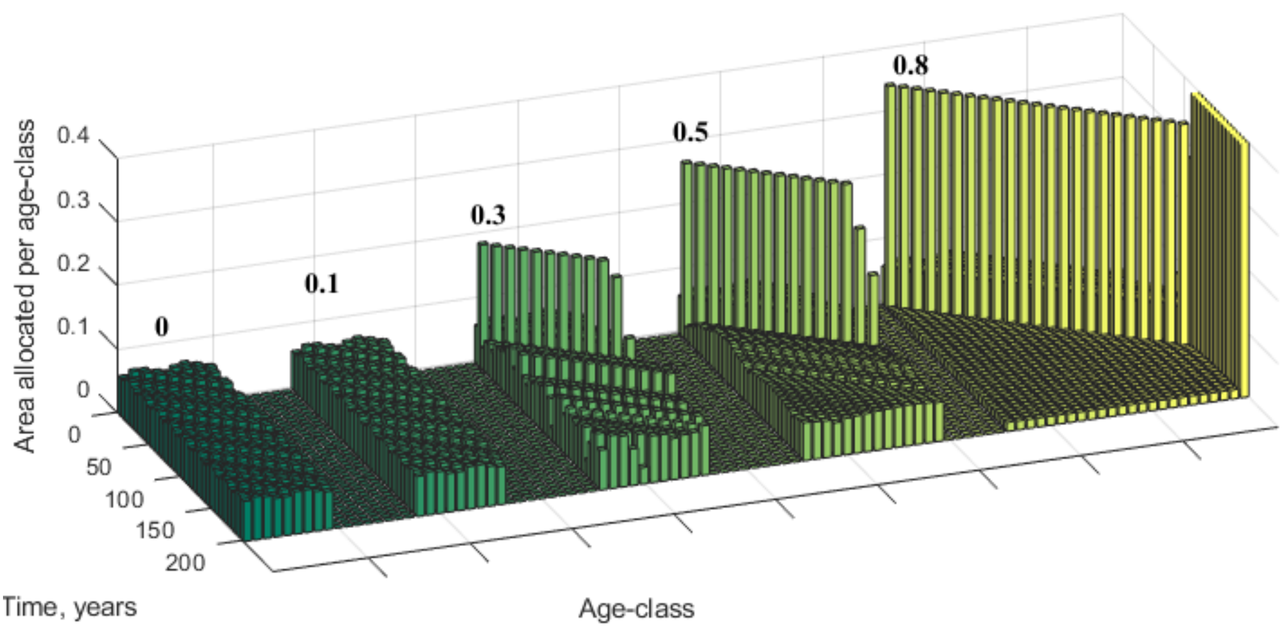


Figure 6. Stationary state harvests when interest rate is 1%.

First-best solutions with carbon policy

Let us analyze an example where a mutual carbon subsidy scheme is applied to both land classes. In Figures 7 a,b it is assumed that r = 0.03, $\tau = 0, 0.1, 0.3, 0.5, 0.8$, $\beta = 1$, and the initial age-class allocation is the equilibrium steady state without carbon subsidies. Carbon subsidies applied to both forestlands result in longer rotation periods, as shown in the stand level analysis, and afforestation, as studied analytically in García et al. (2018). As afforestation is possible only in the first age-class, it results in high cyclicality over the transition period. At a carbon price of 0.1, the cycles smoothen over time, resulting in a new equilibrium stationary state and even total timber flow (Figures 8a,b). Transition to longer rotation periods decreases harvests for the transition period but results in a higher timber supply in the steady state. Higher timber supply leads to a decrease in timber price. When the carbon price is 0.5, the poor forest is allocated solely for carbon storage purposes and fertile forest converges towards normal forest state (Figure 7b). At a carbon price of 0.8, part of the fertile forest is left unharvested and the rest is clearcut at age class n-1, a scenario proven analytically in Tahvonen and Rautiainen (2017).

(a)



(b)

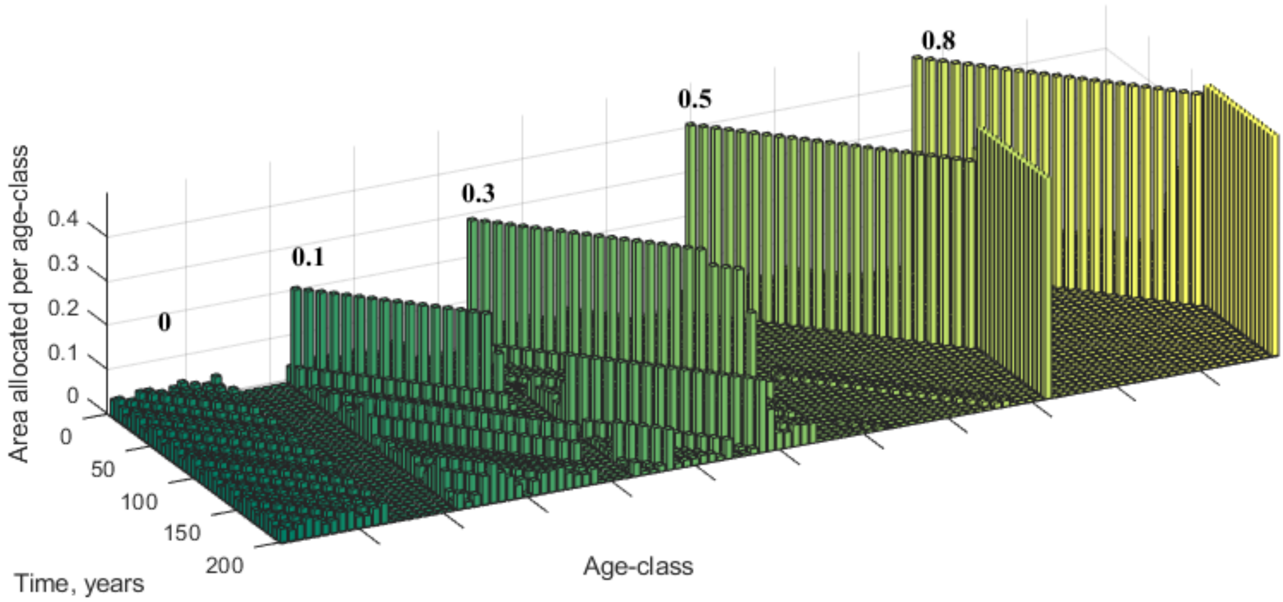
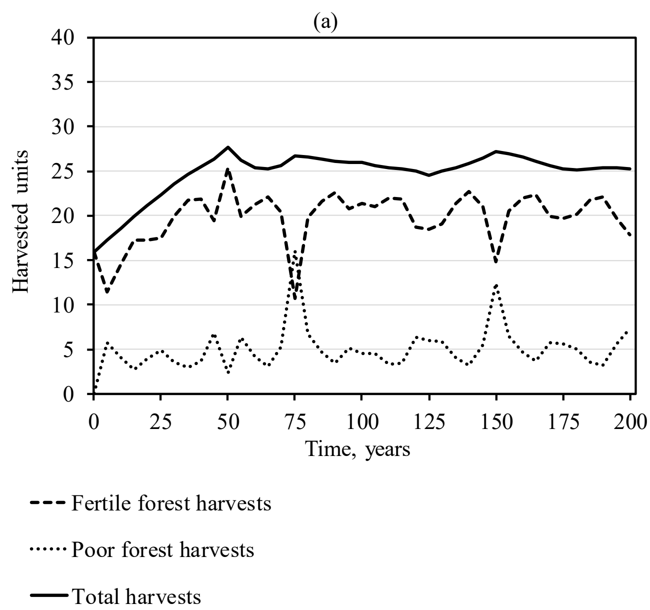
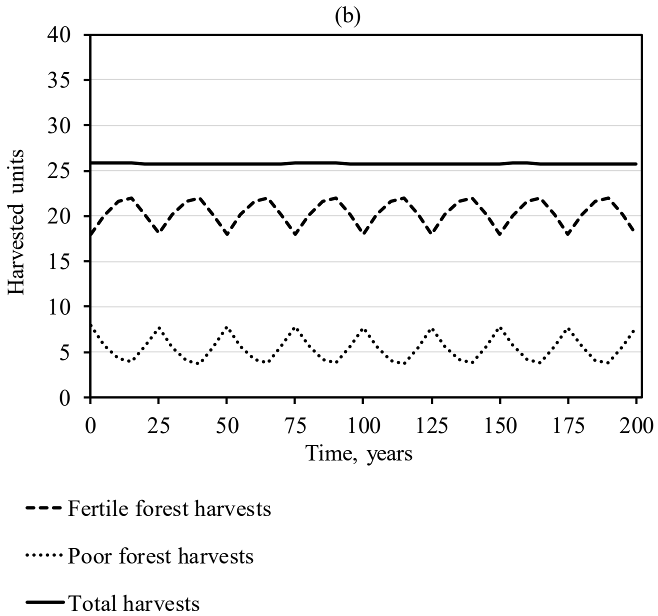


Figure 7a,b. Age-class structure development of (a) fertile and (b) poor forest with different carbon prices.





Figures 8a,b. (a) transition period harvests and (b) stationary state harvests under carbon subsidy. Note. Parameter values are $r = 0.03$, $\tau = 0.1$, and $\beta = 1$.

At a carbon price of 0.3, cycles in the optimal solution does not smoothen and the total harvest converges towards a cyclical stationary state (Figure 9). Computation shows that, with a carbon price of 0.3, the OFF for poor forest is 95 years. Now, with both forests supplying timber to the same markets, poor forest is clearcut at 115 years. This suggests that the stumpage price is set lower by the fertile forest supplying the majority of timber in the market, which lengthens the optimal rotation of the poor forest. The poor forest takes the stumpage price as given and follows the optimal solution almost as if the utility function was linear. When the utility function is linear, each sub-plot of the forestland follows a periodic Faustmann solution as in the stand-level model and does not converge towards a normal forest state (see Mitra and Wan 1985). This explains the large cyclicality of the solution and further supports the notion that cyclicality is a rather fundamental phenomenon in forest economics (cf. Salo & Tahvonen 2004).

Table 2 shows landowner's income under carbon subsidies. The initial age-class allocation is the equilibrium steady state without carbon subsidies and the annual timber production average is approximated from the new equilibrium steady state. The net present value of carbon subsidies is based on additionality (see Tahvonen and Rautiainen 2017). Moderate carbon prices increase the income from forestry due to afforestation and increase timber output. Afforestation leads to a decrease in income from alternative land use. Sufficiently high carbon prices lead to a decrease in the income from forestry as the rotation period lengthen. The effects of the carbon subsidies are stronger in the poor site, as shown in the stand-level analysis. In addition to the stand-level analysis, the market-level example shows that an increase in carbon price may induce afforestation also in forests under conservation (Table 2). Lower interest rate decreases the effects of carbon subsidies. When $\beta < 1$, forestry is more profitable under carbon policy and clearcut is optimal at higher carbon prices.

The cost of an additional unit of CO_2 stored is calculated by comparing the loss of income from both forestry and alternative land use to the increase in the carbon storage (Table 2). In line with the stand-level analysis, the cost of an additional unit of stored CO_2 is higher in the poor forest site until the forest is under conservation. Slower growth rate leads to longer rotation periods and the decrease in timber income is higher due to discounting. After the poor forest is under conservation, an additional unit of stored CO_2 is more expensive in the fertile site due to greater losses in timber income.

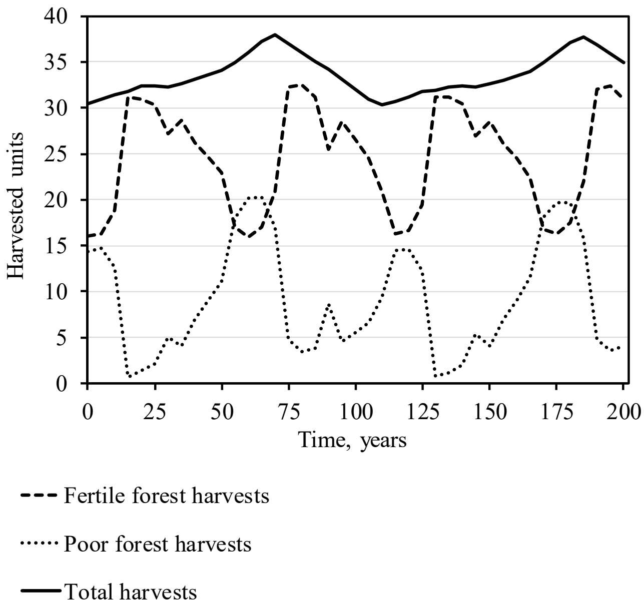


Figure 9. Stationary state harvests with carbon price 0.3.

Table 2. Mutual carbon subsidies in poor and fertile forestland.

	(A)	(B)	(C)	(D)	(E)	(F = C + D + E)	(G)	(H)	(I)	(I)
Site	CO_2 price	Rotation period (years)	NPV timber income(€)	NPV of alternative land use(€)	NPV of carbon subsidies(€)	Total income (€)	Present value of carbon stored (tCO2)	Area allocated to forestry	Annual wood production avg.	Cost of additional CO2 stored (€)
Poor forest	0	65	10,1	6	0	16,1	9,3	0,48	3,6	0
	0,1	75	10,5	5	0,6	16,1	15,1	0,6	5,5	0,1
	0,3	115	7,3	3,3	8,3	18,9	37,1	0,8	10,1	0,19
	0,5	Inf.	0,1	2,2	33,6	35,9	57,8	0,9	0	0,28
	0,8	Inf.	0	1,6	40,5	42,1	60,0	0,95	0	0,29
Fertile forest	0	45	47	25,2	0	72,2	35,7	0,58	17,3	0
	0,1	50	47,8	23,1	1,4	72,3	49,3	0,63	20,3	0,09
	0,3	55	49	17,9	10,3	77,2	70,2	0,75	24,8	0,15
	0,5	70	43,9	13,3	24,3	81,5	102,8	0,85	25,5	0,22
	0,8	115*	15,2	9	106,6	130,8	168,9	0,92	6,7	0,36

Note: Parameter values are r = 0.03, β = 1.
* marks the situation when a part of the forestland is allocated under conservation.

Second-best solutions and carbon leakage

In economics, a theory of the second-best policy concerns a situation when the optimal solution (first-best policy) cannot be applied (Lipsey and Lancaster 1956). In our context, second-best policy would be a unilateral subsidy scheme as the parties fail to form a mutual policy. The market-level model allows us to study the effects of unilateral carbon policy outside the policy area, which can be compared to previous economic studies on carbon leakage based on market-level models which do not consider the dynamics of forest growth (cf. Murray et al. 2004, Gan and McCarl 2007).

García et al. (2018) present the concepts of positive and negative carbon leakage between open-access and commercial forestry. Positive leakage occurs when a unilateral carbon sequestration policy leads to an increase in carbon emissions outside the policy area. If the carbon sequestration policy leads to a decrease in carbon emissions outside the policy area, negative leakage has occurred. Garcia et al. (2018) show analytically how negative leakage may occur between a commercially managed forest, where a carbon policy leads to afforestation, and an open-access forest, where a carbon policy leads to conservation. Note that García et al. (2018) do not consider the case where a sufficiently high carbon price leads to conservation in commercial forestry, as shown in our previous examples.

Let us first study the effect of a unilateral policy between two identical forests. In Figure 10 it is assumed that r = 0.03, $\beta = 1$, and both forests have a high productivity. The initial age-class allocation is the stationary state without carbon subsidies. The bars in Figure 10 show the percentual leakage in the carbon emissions to the outside of the policy area with different carbon prices. The amount of leakage is calculated by comparing the increase in the present value of carbon tons sequestered inside the policy to the decrease in the sequestered carbon outside the policy. Again, we use the present value of carbon tons stored due to the time preference, i.e. the carbon tons sequestered now are valued higher than in the future, as the negative impacts of carbon in the atmosphere accumulate. In our examples, the rate of time preference equals the market interest rate. As shown in Figure 10, moderate carbon prices increase the carbon emissions outside the policy area, i.e. the carbon policy leads to positive carbon leakage. Moderate carbon subsidies lead to afforestation and an increase in timber output inside the policy area, which leads to a decrease in timber price and cause deforestation in other areas providing timber to the same markets. A sufficiently high carbon price leads to forest conservation, which in turn increase the timber price in the markets, inducing afforestation and negative carbon leakage outside the policy area (Figure 10).

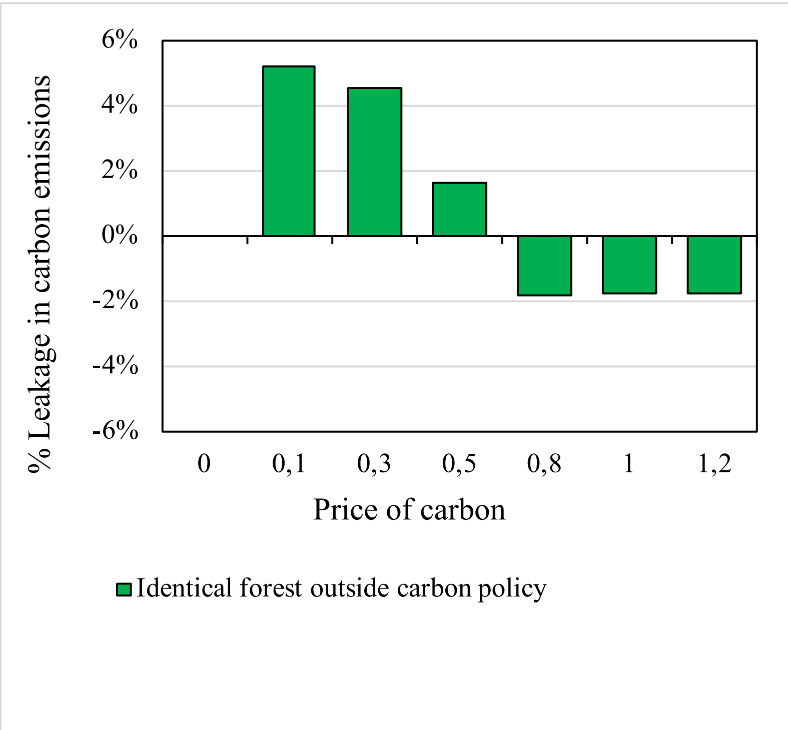


Figure 10. Leakage in carbon emissions due to a unilateral carbon policy.

In Figures 11 a,b it is assumed that r = 0.03, $\beta = 1$, and forests supplying timber to the markets have different productivities. The poor forest reacts more heavily to lower prices of carbon, thus inducing a higher increase in carbon emissions outside the policy area. A higher price of carbon stored in the poor forest lengthen the rotation period so that the positive carbon leakage decreases outside the policy area (Figure 11a). The effect of carbon price is opposite when the fertile site is under a carbon policy: lower prices of carbon have a smaller effect on the optimal forestry, thus inducing less leakage. Higher carbon prices in the fertile forest increase timber output and cause deforestation outside the policy area (Figure 11b). In both examples, a sufficiently high carbon price leads to forest conservation and causes afforestation and negative carbon leakage outside the policy area.

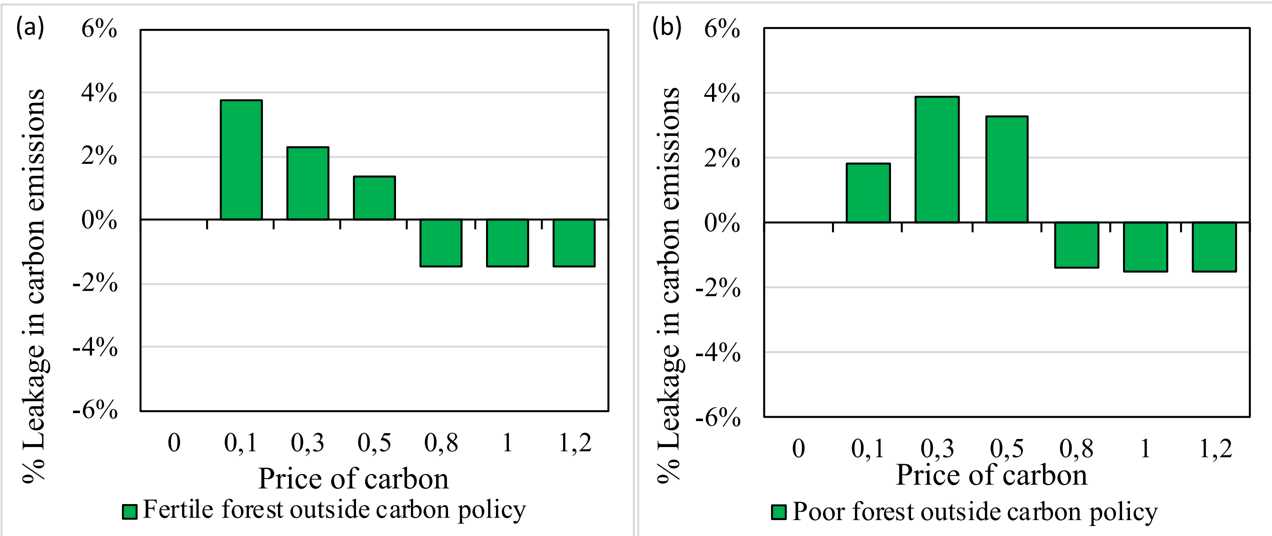
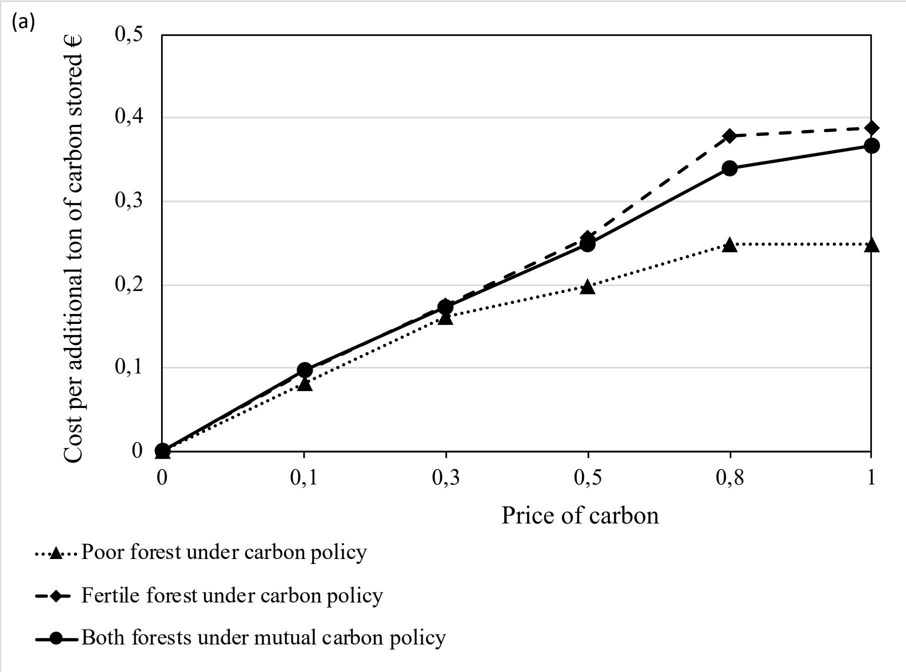
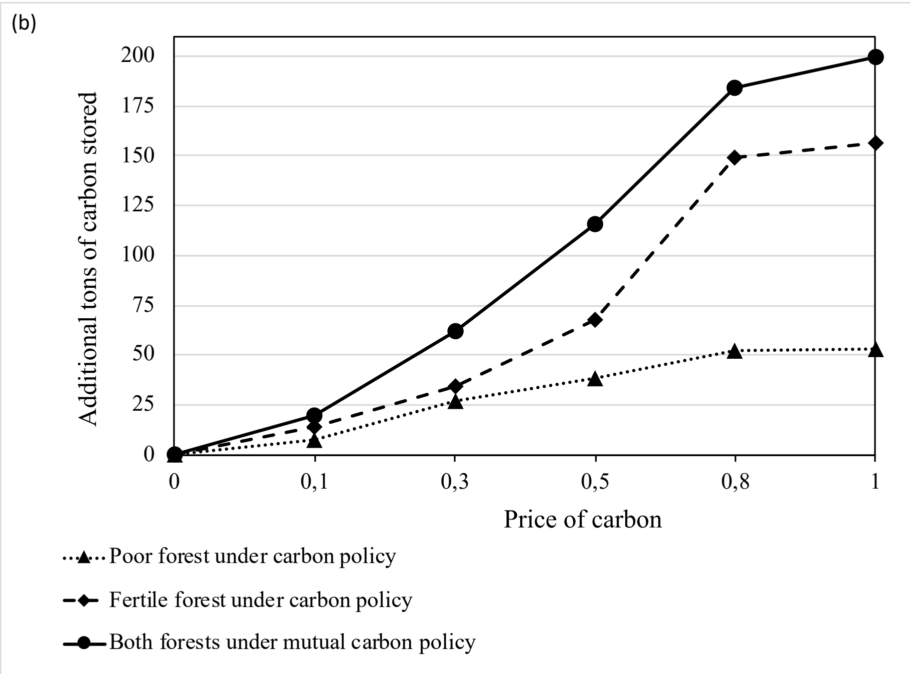


Figure 11 a,b. Leakage in carbon emissions due to a unilateral carbon policy implemented (a) in the poor forest and (b) in the fertile forest.

In contrast with the first-best policy examples, in both fertile and poor forest the optimal management regime changes from clearcut to forest conservation at the same carbon price 0.8. As the fertile forest does not increase its timber supply outside the policy, the timber price stays at a higher lever and it is optimal to clearcut the poor forest with a higher carbon price. Similarly, the timber supply of the poor forest outside the policy decrease the timber price also at the carbon price of 0.8, leaving the fertile forest unharvested earlier than in the first-best policy. Although these results are dependent on the forest growth parameters and the utility function $U(c_t)$ and cannot be generalized as such, it gives us some intuition that the change in the optimal management regime is affected by a) the carbon price, b) the diversity of the forests supplying timber to the markets, and by c) the extent of the carbon policy.

Figures 12 a,b show the cost of an additional ton of sequestered carbon and the amount of additional carbon sequestered in different policy scenarios. The cost of carbon tons is derived from the total losses of income of all land-owners and compared to the total amount of additional carbon sequestered. Intuitively, the amount of sequestered carbon and the cost per additional ton of carbon increase as the carbon subsidy increases. A mutual carbon policy for both forestlands leads to the highest amount of additional carbon sequestered. A unilateral policy implemented on the poor forest leads to the smallest cost and to the smallest amount of carbon sequestered. The costs are the highest when the fertile forest is under a unilateral carbon policy. The results suggest that the cost of an additional unit of carbon stored is dependent on the total amount of carbon sequestered and on the extent of the carbon policy.





Figures 12 a,b. (a) The cost of an additional ton of carbon sequestered and (b) the amount of additional carbon sequestered in different policy scenarios.

Note. Parameter values are $r = 0.03$, and $\beta = 1$.

4 Discussion

The numerical examples of the stand-level model yield results that are in line with the previous economic literature. A positive carbon price lengthens the rotation period when the interest rate is positive and decay rate is one. A high enough carbon price leads to an optimal solution where the stand is not clearcut at all. Lower decay rate shortens the optimal rotation period and increases the income from subsidies. Carbon subsidies can be very costly to the society and should be based on additionality, as presented in Tahvonen & Rautiainen (2017). A moderate carbon price may increase the annual timber output of a forest stand, implying that the optimal rotation without carbon subsidies is not at the MSY level. Average cost per additional unit of CO_2 stored is lower at the high productivity forest stand until the point where the stand of poor productivity is allocated solely for carbon storage purposes. These results are in line with Pihlainen et al. (2014) but contradicts with Niinimäki et al. (2013), where the less fertile site was more cost-efficient with moderate carbon prices. The model in Niinimäki et al. (2013) include thinnings and show that the role of thinnings are stronger at the poor site. All the effects of carbon subsidies on the optimal solution are stronger in the poor forest stand.

The issue with stand-level analysis and exogenous timber price is that it does not take into account market-level implications when carbon subsidies change the optimal management regime. Numerical results of the market-level model shows that moderate carbon prices lengthen rotation periods and increase timber price for the transition period regardless of the policy being unilateral or applied to both land classes. After the transition, total timber supply remains higher and timber price settles at a lower level. With a sufficiently high carbon price, forest is left under conservation as in the stand-level examples. Again, all the effects of carbon subsidies on the optimal solution are stronger in the poor forest. The average cost per additional CO_2 unit stored is higher in the poor forest until the forest is allocated under conservation, as in the stand-level examples.

By studying the change in the net present value of carbon net flow, we obtain examples of carbon leakage when the carbon policy is unilateral. The amount of occurred leakage in our examples is considerably lower than in the previous studies (e.g. 42%-95% estimated positive leakage in Gan and McCarl 2007). Partly the large difference is due to independent alternative land use in our model. If alternative land use would compete in the same markets (e.g. same agricultural crop), it would certainly have an effect on the carbon leakage, as carbon policy may induce afforestation. However, in some cases it is well justified to assume that alternative land use is independent between land classes. Our growth parameters are estimated from Southern Finland (fertile site) and the very North of Finland (poor site). These areas would have very different alternative land uses even though the timber production competes in the same market, as the growth conditions and growth time differ.

Some previous studies (e.g. Murray et al. 2004, Gan and McCarl 2007) on carbon leakage consider timber production as an indicator of leakage instead of the change in carbon net flow. This premise ignores the effects of afforestation on carbon leakage. Our results show that, as unilateral policy leads to forest conservation with a sufficiently high carbon price, increase in the timber production in other areas leads to afforestation and negative carbon leakage in commercially managed forests. However, it is notable that in the case of an open-access forest, increase in the timber supply would inarguably lead to positive carbon leakage, as no-one has the incentive to regenerate the forest (García et al. 2018). It is also notable that as carbon sequestration program may increase long-term harvests, it may replace harvests that are more unsustainable and possibly illegal. Change in the timber price may also affect the market share of other materials, which is outside of boundaries of our analysis.

Mathematical analysis by Murray et al. (2004) argue that small carbon sequestration projects have relatively higher carbon leakage due to the higher elasticity in the demand. Our results suggest that the amount of carbon leakage is similar regardless of the harvest volume of the forest under carbon policy. As Murray et al. (2004) consider sole timber production as indicator of leakage, it is also static and omits the time factor completely. It is possible that our model would give similar results if we would place a forest under carbon policy at time *t* and analyze the immediate changes in carbon net flow. However, given that time is an essential feature in climate change and forest economics, it is well justified to study long-term changes in carbon net flow.

Time is an essential factor when considering the increase in timber output under carbon policy. Both stand-level and market-level examples show that, with moderate carbon prices, the forest under carbon policy increase its long-term timber supply. However, as carbon prices lengthen the optimal rotation, short-term timber output decreases due to carbon subsidies. An immediate reaction to a decrease in the timber output would be an increase in timber price, which would lead to increasing harvests outside the policy area. This explains why choosing timber output as indicator of leakage without considering the policy's long-term effects on carbon sequestration may result in an increase in carbon emissions outside the policy area. However, given the long-term nature of the costs caused by excessive carbon dioxide in the atmosphere, we can argue that there is a fundamental difference between evaluating short-term shifts in the markets and including both the short- and the long-term effects of the policies under investigation.

It is noteworthy that the change in the optimal management regime due to carbon subsidies depends on the forest productivity. Our stand-level examples suggest that a forest of poor productivity is placed under conservation with considerably low carbon prices, which may induce afforestation outside the policy area. Afforestation outside the policy area would result in further decrease in carbon emissions, which would enhance the effects of the sequestration program. In the market-level second-best policy analysis, our results suggest that the change in the management regime would be optimal at the same carbon price in both forests. This implies that the extent of the carbon sequestration program has an effect on the management regime change, as the results differ from the first-best policy scenario. The difference between stand-level and market-level results can be explained by the mutual stumpage price in the market-level model, as the forests face different prices in the stand-level example.

The longer rotation periods due to carbon subsidies may also lead to changes in sawlog and pulpwood supply, which would have market effects of its own. The effects that carbon subsidies have on the demand side is a topic that should be investigated in the future research. As we study forests with different productivities, it should be noted that there are various services forests provide other than sole timber production. Forest with poor productivity may provide other ecosystem services so that the economic viability of sole timber production is nonexistent. Amenity values, value of biodiversity and forest's value for tourism are factors that should be considered in the future market-level studies as well.

In our market-level model, poor and fertile forestland have a mutual stumpage price. Often the poor forestlands suffer from long distances and inaccessibility, resulting in higher costs and smaller stumpage price. This could be achieved by including a coefficient for logistics in the poor forestland harvests. Intuitively, this would increase the effects of carbon subsidy program and support our results further. In addition, adding regeneration cost into the model could lengthen the rotation periods and enhance the effect of negative leakage. Even though the economic models used in this thesis has assumptions and simplifications of its own, it does provide a sound argument that carbon sequestration programs in forestry does not necessarily lead to positive carbon leakage.

5 Conclusions

The purpose of this thesis is to show the effects of carbon subsidies on forests of different productivities in stand-level analysis and in market-level analysis. The stand-level analysis is studied with the Faustmann model (1849) with carbon storage, as presented by Van Kooten et al. (1995). The market-level analysis expands the age-class structured model by Mitra and Wan (1985) with multiple land classes, as in Salo & Tahvonen (2002) and alternative land use and carbon storage, as presented by Tahvonen & Rautiainen (2017). The numerical examples of both analyses show that carbon subsidies may lengthen the optimal rotation period, increase the annual timber output and increase the amount of carbon units stored in the forest. A high enough carbon price leads to the entire forest or a part of the forest left unharvested. All the effects are stronger in the forest of poor productivity.

The market-level results show that, with two land classes and alternative land use, the forestland-specific harvests are cyclical and lead to an even total timber flow. Carbon subsidies lengthen the optimal rotation, induce afforestation and increase the timber output. An increase in the timber output leads to a decrease in the market price. A sufficiently high carbon price leads to forest conservation, as in the stand-level example. All the effects of carbon subsidies are stronger in the poor forest.

Unilateral policies lead to increase in timber output inside the policy, which decreases the timber price and results in deforestation outside the policy area. As sufficiently high carbon price leads to forest conservation, timber price increases and results in afforestation and negative carbon leakage outside the policy area. Maximum amount of leakage is similar in both forests and the magnitude of leakage depends on the carbon price. The results are in contradiction with the common hypothesis that increasing carbon sequestration in forests by unilateral policy would inevitably lead to an increase in carbon emissions outside the policy area.

References

Akao, K.-I., 2011. Optimum forest program when the carbon sequestration service of a forest has value. Environ. Econ. Policy Stud. 13, 323–343.

Assmuth, A., Rämö, J., & Tahvonen, O. 2018. Economics of size-structured forestry with carbon storage. Canadian Journal of Forest Research, 48(1), 11–22.

Baumol, W. 1972. On Taxation and the Control of Externalities. The American Economic Review, 62(3), 307-322.

Cai, Y., Judd, K., and Lontzek, T., 2015. The social cost of carbon with economic and climate risks. Journal of Political economy.

Caparrós, A., Campos, P., and Martín, D. 2003. Influence of carbon dioxide abatement and recreational services on optimal forest rotation. International Journal of Sustainable Development, 6(3): 345–358.

Chow, G. C., 1997. Dynamic Economics: Optimization by the Lagrange Method. Oxford Univ. Press, New York.

Cunha-e-Sá, M., Rosa, R., 2006. The Impact of Carbon Accounting Methods on Optimal Forest Management: An Application to the Portuguese Eucalyptus Forest.

Cunha-e-Sá, M.A., Rosa, R., Costa-Duarte, C., 2013. Natural carbon capture and storage (NCCS): forests, land use and carbon accounting. Resour. EnergyEcon. 35, 148–170

The European parliament and the council. 2018. Regulation 2018/841 (EU 2018/841). Official Journal of the European Union.
https://eurlex.europa.eu/legalcontent/EN/TXT/PDF/?uri=CELEX:32018R0841&from=EN

Faustmann, M. 1849. Berechnung des Werthes, welchen Waldboden, sowie noch nicht haubare Holzbestände für die Waldwirthschaft besitzen. Allgemeine Forst- und Jagd-Zeitung 25, 441–455.

Gan, Jianbang; McCarl, Bruce, 2007. Measuring transnational leakage of forest conservation, Ecological Economics, vol. 64, pp. 423-432.

García, J. H., Orlov, A., & Aaheim, A. 2018. Negative leakage: The key role of forest management regimes. Journal of Forest Economics, 33, 8–13.

Gong, P., and Kriström, B. 1999. Regulating forest rotation to increase CO2 sequestration. SLU Inst. Skogsekonomi, Arbetsrapport 272, pp. 1–22.

Hagmann, D., Ho, E. H., & Loewenstein, G. 2019. Nudging out support for a carbon tax. Nature Climate Change, 9(6), 484–489.

Harstad, B., Mideksa, T.K., 2017. Conservation contracts and political regimes. Rev. Econ. Stud. 84 (4), 1708–1734.

Hartman, R. 1976. The Harvesting Decision When a Standing Forest Has Value. Economic Inquiry 14, 52–55.

Hoen, H. F., & Solberg, B. 1997. CO2-taxing, timber rotations, and market implications, Critical Reviews in Environmental Science and Technology, 27:S1, 151-162.

IPCC. 2019: Land–climate interactions. In: Climate Change and Land: an IPCC special report on climate change, desertification, land degradation, sustainable land management, food security, and greenhouse gas fluxes in terrestrial ecosystems [P.R. Shukla, J. Skea, E. Calvo Buendia, V. Masson-Delmotte, H.-O. Pörtner, D.C. Roberts, P. Zhai, R. Slade, S. Connors, R. van Diemen, M. Ferrat, E. Haughey, S. Luz, S. Neogi, M. Pathak, J. Petzold, J. Portugal Pereira, P. Vyas, E. Huntley, K. Kissick, M, Belkacemi, J. Malley, (eds.)]. In press.

Kaipainen, T., Liski, J., Pussinen, A., and Karjalainen, T. 2004. Managing carbon sinks by changing rotation length in European forests. Environ. Sci. Policy, 7(3): 205–219.

Kallio, M., & Solberg, B., 2018: Leakage of forest harvest changes in a small open economy: case Norway, Scandinavian Journal of Forest Research.

Kaplan, W., 1973. Integrals Depending on a Parameter—Leibnitz's Rule. Advanced Calculus (2nd ed.). Reading: Addison-Wesley. pp. 285–288.

Lipsey, R., & Lancaster, K., 1956. The General Theory of Second Best. Review of Economic Studies. 24. 11-32. 10.2307/2296233.

Liski, J., Pussinen, A., Pingoud, K., Mäkipää, R., and Karjalainen, T., 2001. Which rotation length is favourable to carbon sequestration? Can. J. For. Res. 31(11): 2004–2013.

Market Insider, 2019. CO2 EUROPEAN EMISSION ALLOWANCES. Accessed 13 November 2019. https://markets.businessinsider.com/commodities/co2-european-emission-allowances

Mitra, T., Wan, H.Y., 1985. Some theoretical results on the economics of forestry. Rev. Econ. Stud. LII, 263–282.

Mitra, T., and H.W. Wan. 1986. On the Faustmann solution to the forest management problem. J. Econ. Theory 40:229–249.

Murray, B., McCarl, B., Lee, H., 2004. Estimating Leakage From Carbon Sequestration Programs, Land Economics, vol. 80, pp. 109-124.

Nepal, P., Ince, P.J., Skog, K.E., Chang, S.J., 2013. Forest carbon benefits, costs and leakage effects of carbon reserve scenarios in the United States. J. For. Econ. 19 (3), 286-306.

Niinimäki, S., Tahvonen, O., Mäkelä, A., Linkosalo, T., 2013. On the economics of Norway spruce stands and carbon storage. Can. J. For. Res. 43 (7), 637–648.

Nordhaus, W. D. 2017. Revisiting the social cost of carbon. Proceedings of the National Academy of Sciences, 114 (7), 1518–1523.

Nurmi,V., Ollikainen, M. 2019. Kohti hiilipörssiä? Ympäristöministeriön julkaisuja 2019:17. Ympäristöministeriö, Helsinki 2019.

Olschewski, R., Benítez, P.C., 2010. Optimizing joint production of timber and carbon sequestration of afforestation projects. J. For. Econ. 16 (1), 1–10.

Piazza, A., Roy, S., 2015. Deforestation and optimal management. J. Econ. Dyn. Control 53, 15–27.

Pihlainen, S., Tahvonen, O., & Niinimäki, S., 2014. The economics of timber and bioenergy production and carbon storage in Scots pine stands. Canadian Journal of Forest Research, 44(9), 1091–1102.

Pohjola, J., Valsta, L., 2007. Carbon credits and management of Scots pine and Norway spruce stands in Finland. Forest Policy Econ. 9 (7), 789–798.

Salo, S., Tahvonen, O., 2002. On equilibrium cycles and normal forests in optimal harvesting of tree vintages. J. Environ. Econ. Manag. 44, 1–22.

Salo, S., Tahvonen, O., 2004. Renewable resources with endogenous age classes and allocation of land. Am. J. Agric. Econ. 86 (2), 513–530.

Samuelson, Paul A. (1976), Economics of Forestry in an Evolving Society. Economic Inquiry XIV, 466-492.

Sedjo, R.A., and K.S. Lyon. 1990. The long-term adequacy of world timber supply. Resources for the Future, Washington, DC.

Stiglitz, J., Stern, N., Duan, M., Edenhofer, O., Gireaud, G., Heal, G., la Rovere, E., Morris, A., Moyer, E., Pangestu, M., Shukla, P., Sokona, Y., Winkler, H., 2017. Report of the High-Level Commission on Carbon Prices. World Bank. https://static1.squarespace.com/static/54ff9c5ce4b0a53decccfb4c/t/59b7f2409f8dce5316811916/1505227332748/CarbonPricing_FullReport.pdf

Sohngen, B., Mendelsohn, R., Sedjo, R., 1999. Forest management, conservation, and global timber markets. American Journal of Agricultural Economics 81 (1), 1–13.

Strang, W. 1983. On the Optimal Forest Harvesting Decision. Economic Inquiry 21, 576–583.

Tahvonen, O., & Rautiainen, A. 2017. Economics of forest carbon storage and the additionality principle. Resource and Energy Economics, 50, 124–134.

Van Kooten, G.C., Binkley, C.S., and Delcourt, G. 1995. Effect of carbon taxes and subsidies on optimal forest rotation age and supply of carbon services. Am. J. Agric. Econ. 77: 365–374.

World Bank, 2019. Pricing Carbon. Accessed 4 December 2019. https://www.worldbank.org/en/programs/pricing-carbon

Appendices

Appendix 1. AMPL/Knitro code for market-level optimization problem.

Parameters and variables

```
param p:=0.3                     #carbon price
param beta:=1                    #carbon decay rate
param T:=400;                    #max time
param r:=0.03;                   #interest rate
param b := 1/(1+r)               #Discount factor
param n=24;                      #max age class
param alpha:=0.8;                #forest demand function parameter
Param gamma:=0.4;                #agricultural land demand function parameter
param W1:=4;                     #Agricultural land utility function
param W2=2;                      #Agricultural land utility function
param U=1;                       #forest utility function
param x10 {s in 1..n};           #forest 1 age class allocation
param x20 {i in 1..n};           #forest 2 age class allocation
param f1:=                       #Fertile forest growth
    1 0
    2 0
    3 0
    4 7
    5 30
    6 76
    7 139
    8 206
    9 269
    10 321
    11 362
    12 393
    13 415
    14 431
    15 442
    16 449
    17 455
    18 458
    19 461
    20 462
    21 463
    22 464
    23 465
    24 465;

param f2:=                       #Poor forest growth
1 0
2 0
3 0
4 1
5 3
6 7
7 13
8 22
9 34
10 48
11 64
12 81
13 99
14 116
15 134
16 151
```

```
17 167
18 182
19 196
20 208
21 220
22 239
23 247
24 254;


var x1 {s in 1..n, t in 0..T+1}>=0;    #forest 1 age class
var x2 {i in 1..n, t in 0..T+1}>=0;    #forest 2 age class
var y1 {t in 0..T+1}>=0;               #agriculture1 land allocation
var y2 {t in 0..T+1}>=0;               #agriculture2 land allocation
var Y {t in 0..T+1}=y1[t]+y2[t];       #total agland amount
var z1 {t in 0..T-1}>=0;               #timber harvest 1
var z2 {t in 0..T-1}>=0;               #timber harvest 2
var h1 {t in 0..T-1}>=0;               #forest1 land allocation
var h2 {t in 0..T-1}>=0;               #forest2 land allocation
var c {t in 0..T-1}=(z1[t]+z2[t]);     #total harvestable timber amount
var woodprice {t in 0..T-2}=U*alpha*(c[t])^(alpha-1); #wood price
var q1 { t in 0..T-1}=sum{s in 1..n} ((f1[s]*(x1[s,t+1]-x1[s,t])+(1-beta)*z1[t])); #carbon net flow in forest 1
var q2 { t in 0..T-1}=sum{i in 1..n} ((f2[i]*(x2[i,t+1]-x2[i,t])+(1-beta)*z2[t])); #carbon net flow in forest 2
var v1 {t in 0..T-1}>=0;               #volume of living trees in x1
var v2 {t in 0..T-1}>=0;               #volume of living trees in x2
var V {t in 0..T-1}=(v1[t]+v2[t]);     #total volume of living trees
var NPV1 {t in 0..T-1}>=-100000; #forest sector net present utility
var ANPV1 {t in 0..T-1}=b^((t+1)*5)*(U*(c[t])^alpha);
var BLV1 {t in 0..T-1}>=-100000;
var ABLV1 {t in 0..T-1}=b^((t+1)*5)*((U*alpha*(c[t])^(alpha-1))*z1[t]); #forest 1 net present utility
var BLV2 {t in 0..T-1}>=-100000;
var ABLV2 {t in 0..T-1}=b^((t+1)*5)*((U*alpha*(c[t])^(alpha-1))*z2[t]); #forest 2 net present utility
var BLV1ha {t in 0..T-1}>=-100000;
var ABLV1ha {t in 0..T-1}=b^((t+1)*5)*((U*alpha*(c[t])^(alpha-1))*z1[t]/(h1[t])); #forest 1 net present utility per ha
var BLV2ha {t in 0..T-1}>=-100000;
var ABLV2ha {t in 0..T-1}=b^((t+1)*5)*((U*alpha*(c[t])^(alpha-1))*z2[t]/(h2[t])); #forest 2 net present utility per ha
var NPVhiili1 {t in 0..T-1}>=-100000;
var ANPVhiili1 {t in 0..T-1}=b^((t+1)*5)*(p*q1[t]); #Carbon subsidy npv
var AgNPV1 {t in 0..T-1}>=-100000;
var AAgNPV1 {t in 0..T-1}=b^((t+1)*5)*((W1*alpha*(y1[t])^(alpha-1))*y1[t]); #agland 1 net present utility
var AgNPV2 {t in 0..T-1}>=-100000;
var AAgNPV2 {t in 0..T-1}=b^((t+1)*5)*((W2*alpha*(y2[t])^(alpha-1))*y2[t]); #agland 1 net present utility
var NPVhiili2 {t in 0..T-1}>=-100000;
var ANPVhiili2 {t in 0..T-1}=b^((t+1)*5)*(p*q2[t]); #Carbon subsidy npv
var Qnpv1 {t in 0..T-1}>=-100000;
var AQnpv1 {t in 0..T-1}=b^((t+1)*5)*(q1[t]);        #npv of carbon net flow 1
var Qnpv2 {t in 0..T-1}>=-100000;
var AQnpv2 {t in 0..T-1}=b^((t+1)*5)*(q2[t]);        #npv of carbon net flow 2
```

Code

```
#Objective function:

maximize objective_function: sum{t in 0..T-2}
b^((t+1)*5)*(U*(z1[t]+z2[t])^alpha+W1*(y1[t])^gamma+W2*(y2[t])^gamma+p*q1[t]+p*q2[t]);

#constraints:

subject to const1{s in 1..n-2, t in 0..T-1}: x1[s+1,t+1]-x1[s,t]<=0;

subject to const2{i in 1..n-2, t in 0..T-1}: x2[i+1,t+1]-x2[i,t]<=0;

subject to const3{t in 0..T-1}: x1[n,t+1]-x1[n,t]-x1[n-1,t]<=0;

subject to const4{t in 0..T-1}: x2[n,t+1]-x2[n,t]-x2[n-1,t]<=0;
```

```
subject to const5{s in 1..n}: x1[s,0]=x10[s];

subject to const6{i in 1..n}: x2[i,0]=x20[i];

subject to const7{t in 0..T-1}: y1[t]=1-sum{s in 1..n} x1[s,t];

subject to const8{t in 0..T-1}: y2[t]=1-sum{i in 1..n} x2[i,t];

subject to const10{t in 0..T-1}: z1[t]=sum{s in 1..n-2} (f1[s]*(x1[s,t]-x1[s+1,t+1]))+f1[n]*(x1[n,t]+x1[n-1,t]-x1[n,t+1]);

subject to const11{t in 0..T-1}: z2[t]=sum{i in 1..n-2} (f2[i]*(x2[i,t]-x2[i+1,t+1]))+f2[n]*(x2[n,t]+x2[n-1,t]-x2[n,t+1]);

subject to const15{t in 0..T-1}: h1[t]=sum{s in 1..n} x1[s,t];

subject to const16{t in 0..T-1}: h2[t]=sum{i in 1..n} x2[i,t];

subject to const17{t in 0..T-1}: v1[t]=sum{s in 1..n} f1[s]*x1[s,t];

subject to const18{t in 0..T-1}: v2[t]=sum{i in 1..n} f2[i]*x2[i,t];
```